

## **Final Report**

### **Theoretical Investigation of Ultracold Few-Body Collisions**

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14. ABSTRACT <p>The general goals of this project were to extend previous theoretical studies of ultracold three-body collisions to new regimes and to identify the limits of the universal behavior of such collisions. Central to the universality of ultracold three-body collisions is Efimov physics. The experimental observation of Efimov physics itself has been a goal for nearly four decades. To aid in this effort, new manifestations of Efimov physics were identified and the practical issues complicating the experimental identification of Efimov physics were examined. Moreover, a scheme for producing Efimov trimers via four-body recombination was proposed and the corresponding rate calculated (the first such calculation of its kind). It was also shown that weakly bound molecules could have a surprisingly long collisional lifetime even in the absence of Fermi statistics, that is, in the absence of identical fermions. Finally, the most realistic calculation of three-body recombination to date for three helium atoms was performed, showing that retardation can be very important for ultracold collisions.</p>					
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**Abstract:**

The general goals of this project were to extend previous theoretical studies of ultracold three-body collisions to new regimes and to identify the limits of the universal behavior of such collisions. Central to the universality of ultracold three-body collisions is Efimov physics. The experimental observation of Efimov physics itself has been a goal for nearly four decades. To aid in this effort, new manifestations of Efimov physics were identified and the practical issues complicating the experimental identification of Efimov physics were examined. Moreover, a scheme for producing Efimov trimers via four-body recombination was proposed and the corresponding rate calculated (the first such calculation of its kind). It was also shown that weakly bound molecules could have a surprisingly long collisional lifetime even in the absence of Fermi statistics, that is, in the absence of identical fermions. Finally, the most realistic calculation of three-body recombination to date for three helium atoms was performed, showing that retardation can be very important for ultracold collisions.

**Publications and Summary of Major Accomplishments:**

All of the major accomplishments are described in the publications produced during this grant period. That being the case, the best summary of each is provided by their respective abstracts which are collected below along with the full reference for each publication.

- “Efimov trimer formation via ultracold four-body recombination,”  
Y. Wang and B.D. Esry,  
Phys. Rev. Lett. **102**, 133201 (2009).

We discuss the collisional formation of Efimov trimers via ultracold four-body recombination. In particular, we consider the reaction  $A + A + A + B \rightarrow A_3 + B$  with  $A$  and  $B$  ultracold atoms. We obtain expressions for the four-body recombination rate up to an overall constant and show that it reflects the three-body Efimov physics either as a function of collision energy or as a function of the two-body  $s$ -wave scattering length between  $A$  atoms. In addition, we briefly discuss issues important for experimentally observing this interesting and unexplored process.

- “The three-body short-range phase and other issues impacting the observation of Efimov physics in ultracold quantum gases,”  
J.P. D’Incao, C.H. Greene, and B.D. Esry,  
J. Phys. B **42**, 044016 (2009).

We discuss several issues important for experimentally observing Efimov physics in ultracold quantum gases. By numerically solving the three-boson Schrödinger equation over a broad range of scattering lengths and energies, and by including model potentials with multiple bound states, we address the complications of relating experimental observations to available analytic expressions. These more realistic potentials introduce features that can mask the predicted Efimov physics at small scattering lengths. They also allow us to verify that positive and negative scattering lengths are universally connected only across a pole, not across a zero. Additionally, we show that the spacing between Efimov features for the relatively small scattering lengths accessible experimentally fails to precisely follow the geometric progression expected for Efimov physics. Finally, we emphasize the importance of the short-range three-body physics

in determining the position of Efimov features and show that theoretically reproducing two-body physics is not generally sufficient to predict three-body properties quantitatively.

- “Suppression of molecular decay in ultracold gases without Fermi statistics,”  
J.P. D’Incao and B.D. Esry,  
Phys. Rev. Lett. **100**, 163201 (2008).

We study inelastic processes for ultracold three-body systems in which only one interaction is resonant. We show that at ultracold temperatures three-body recombination in such systems leads mainly to the formation of weakly bound molecules. In addition, and perhaps more importantly, we have found that the decay rates for weakly bound molecules due to collisions with other atoms can be suppressed not only without fermionic statistics but also when bosonic statistics applies. These results indicate that recombination in three-component atomic gases can be used as an efficient mechanism for molecular formation, allowing the achievement of high molecular densities.

- “Ultracold atom-molecule collisions with fermionic atoms,”  
J.P. D’Incao, B.D. Esry, and C.H. Greene,  
Phys. Rev. A **77**, 052709 (2008).

Elastic and inelastic properties of weakly bound  $s$ - and  $p$ -wave molecules of fermionic atoms that collide with a third atom are investigated. Analysis of calculated collisional properties of  $s$ -wave dimers of fermions in different spin states permit us to compare and highlight the physical mechanisms that determine the stability of  $s$ -wave and  $p$ -wave molecules. In contrast to  $s$ -wave molecules, the collisional properties of  $p$ -wave molecules are found to be largely insensitive to variations of the  $p$ -wave scattering length, and these collisions will usually result in short molecular lifetimes. We also discuss the importance of this result for both theories and experiments involving degenerate Fermi gases.

- “Alternative paths to observing Efimov physics,”  
B.D. Esry, Y. Wang, and J.P. D’Incao,  
Few-Body Syst., **43**, 63 (2008).

We present some of our work on the ultracold behavior of three-body collisions and their relation to the recent experiment [reported in] T. Kraemer *et al.*, Nature **440**, 315 (2006). In particular, we discuss the role of Efimov physics in this experiment and other ultracold three-body collisions. We also suggest one way to make observation of the key feature of the Efimov effect — geometrical scaling — more experimentally feasible.

- “Adiabatic hyperspherical study of triatomic helium systems,”  
H. Suno and B.D. Esry,  
Phys. Rev. A **78**, 062701 (2008).

The  $^4\text{He}_3$  system is studied using the adiabatic hyperspherical representation. We adopt the current state-of-the-art helium interaction potential including retardation

and the nonadditive three-body term to calculate all low-energy properties of the tri-atomic  $^4\text{He}$  system. The bound state energies of the  $^4\text{He}$  trimer are computed as well as the  $^4\text{He}+^4\text{He}_2$  elastic scattering cross sections, the three-body recombination, and collision induced dissociation rates at finite temperatures. We also treat the system that consists of two  $^4\text{He}$  and one  $^3\text{He}$  atoms, and compute the spectrum of the isotopic trimer  $^4\text{He}_2^3\text{He}$ , the  $^3\text{He}+^4\text{He}_2$  elastic scattering cross sections, the rates for three-body recombination, and the collision induced dissociation rate at finite temperatures. The effects of retardation and the nonadditive three-body term are investigated. Retardation is found to be significant in some cases, while the three-body term plays only a minor role for these systems.

#### Personnel involved:

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Faculty:	Brett D. Esry (PI)
Graduate students:	Erin Lynch, Dec. 2006 – Dec. 2008

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This grant supported only a single graduate student, Erin Lynch. Unfortunately, during her time here, she had several health problems that prevented her from making substantial progress. These problems led to her switching from being supported by this grant back to a teaching assistantship at different times during the period indicated above, and ultimately to leaving my group and our graduate program without a degree. Thus, Erin did not finally contribute sufficiently to appear on any publication from my group. Without another student to take her position, the grant was used to partially support me in the summers. The publications listed above are the result of that partial support. The other authors on these publications were supported by separate grants.